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NURBS IN ANIMATED ARCHITECTURE

This paper presents the relations between terms in Foucault's philosophy and terms in computer graphics. The attention is particularly focused on curves in Foucault's description of relations in diagram and on NURBS splines in animation software.
The main goal of this paper is to show why NURBS geometries have become so important for architects, who are using animation concepts in their works.

Introduction

As a thesis researcher of the subject Animated Forms I was invited by Imro Vaško to become one of the advisors of a summer term project at the Laboratory of Architecture at the Academy of Art. The students were supposed to explore fluid spaces and blob-kind structures. I, as a researcher of animated forms, felt quite comfortable in accepting that challenge. It turned out to be slightly more challenging, than I was expecting.
At that time, there were few questions/subjects, that mostly drove my/our interest: the importance of topology, fluid forms, usage of non-Euclidean spaces, animated techniques in architecture and NURBS surfaces. Although I had already been studying that subject for two years, discussions with other students revealed many more questions, and some of them have kept me busy until now. One in particular has been very persistent and has opened up many new views in this area of research for me. It was Juraj Sukop, who asked me that question. He as a former student of mathematics and a good programmer was from the beginning trying to discredit the potential of animated techniques and input of such architects as Gregg Lynn. He asked, why is it so important for Gregg Lynn to work with NURBS surfaces or why it just couldn't be the surface of a mathematic function. I have to admit, that at that time I wasn't really able to answer the question, but as I already said I kept it in mind while researching my thesis and I believe that now I can do so.

At that time my understanding of NURBS was as the best a way to construct smooth curved surfaces in a computer. Greg Lynn has been using it with Maya® and he has been also writing about it, so I believed him, and even more,
I somehow became to think about it as the only possibility how to draw smooth surfaces. I have realized that I have almost completely ignored the possibility of drawing a surface as a result of a given function and that I was also not sure what are the differences between them. I saw the advantage of NURBS in its parametric attribute. I felt that the algorithmic character makes it more suitable for an animated form or that the time value in its algorithm somehow guarantees fluidity and animated attributes of resulting form. Later I believed that its advantage was mainly in the ability to visualize smooth shapes and also to reinterpret/recode the geometry to production machines.
At that time I used to think of NURBS as a more non-Euclideanlike geometry and on the other hand about Á a polygonal geometry as a Cartesian-like geometry. Nurbs as a topological U and V vector based geometry and polygon as a X,Y,Z point based geometry. The surface or spline as a result of a mathematical function was something, that has been overlooked in this dualistic perspective. Neither the algorithmic quality nor the ability of creating perfect smooth geometry persuades me of the importance and inevitability of Gregg Lynn's work.
Fortunately, after further reading Lynn's, Foucault's and Deleuze's books, I found out why NURBS and works of these people are so connected and why the use of this geometry was the best choice for Lynn.

Animation and NURBS

At first thought the term "animated forms" would probably evoke the idea of animated films and films as such. Greg Lynn, a pioneer of animated technique in architecture, implies this connection as well. Educated as an architect and philosopher as well, he uses in his projects animating software, which were primarily developed for special effects within the film industry. Even some of his concepts are inspired by Hollywood bestsellers (for example Predator, The Blob, etc.) These methods can at first sight look less serious and shallow. They can be understood just as an effort to find an uncommon and complicated form.

A deeper investigation regarding the uses of animated methods in architecture, by architects as Greg Lynn and Lars Spuybroek, reveals direct connections to the works of Frei Otto or Antony Gaudi. On the other hand detailed reading of their writings refers to philosophical works of G.Deleuze, M.Foucault and ancient Greece.

If it is possible to summarize the most important aim of these methods, then I would use the word "complexity". Complexity, by means of dragging away from idealized bipolarized meanings. All this in order to be able to work at once with as wide a range of differentiated and integrated variables as possible. Presence or simultaneity is an important condition and at the same time one of the reasons for the use of animated softwares and parametrical geometries.
One of the methods how they are trying to reach the complexity is a work within a space between fixed or specified forms/ terms. It is not anymore a game between two opposites like fullness and emptiness or point and space. The attention is brought to

[1] See page 54

space/non-space between these opposites. The aim is to 'grasp' it, or (using Foucalt's terms) to articulate or visualize this open gap. Blurring, healing, continuity and fluidity become means of expressing of these aims.

The most common tool for grasping this space/non-space has become a diagram.
Architect Peter Eisenman in his book Diagram Diaries divides use of the diagram into two main methods. He describes how a diagram is used by J.Derrida and on the other side by G. Deleuze and M. Foucault. Derrida's diagram can be compared to Freud's 'Mystic Writing Pad' which consists from three layers. The outer one or surface is where the writing takes place, the middle layer transcribes the text from the upper to the lower one, on which the inscription form of the writing is retained. This tool offers endless possibilities of writing and subsequently rewriting on its upper layer and consequently 'recording' many series of superimosed marks into the lower layer.

"The architectural diagram, like Mystic Writing Pad, can be conceived of as a series of surfaces or layers which are both constantly regenerated and at the same time capable of retaining multiple series of traces." [2]

"The diagram understood as a strata of superposed traces offers the possibility of opening up the visible to articuable, to what is within the visible. In this context architecture becomes more than that which is seen or which is present; it is no longer entirely a representation or an illustration of presence. Rather, architecture can be re-presentation of this intervening apparatus called diagram." [3]

Peter Eisenman emphasised[4] that the result of the Derida's diagram is a layer of superimposed inscriptions, in which he consequently finds/concretizes the resulting form. In the opposite, he puts Deleuze's diagram or major stream of architects, theoretician and philosophers that try to build up a diagram so that its function can be visualized. In other words, they materialize an abstract machine.
One of the techniques of its materialization is also animation by using animation software.
Here pops up a question what is Deleuze's diagram.
It is a re-interpretation of Fucautl's diagram as a strength series. It is not structure, something what could be named as hierarchical or static with one firm point towards which it can be referred to. It is an abstract machine, nonformal function that operates on non-formed substance. This diagram is an open system of mutually dependent functions and variables. It is a distribution system of forces that influence and at the same time are being influenced. It is both form and matter and can be visualized and articulated.

The reason of the materialization is in time and force essence of a diagram. Also Foucalt in his work describes/articulates force functions and their relations by using terms from geometry and mathematics. He describes them as spatial curves passing through the points in space. This system of curves and points is non-hierarchical and open. Curves are a spatial visualization of forces impact. Their shape is determined by positions of points and also by weights of points. Weight of point is strength of a force affecting curve in a point's neighbourhood. At the same time curves are representations of forces, which can also affect a point's position. In short, a curve is determined by points and points are determined by curve.

"For Foucault, regularity has a precise meaning: it is the curve joining individual points (a rule). To be precise, the relations between forces determine individual points, such that a diagram is always a transmission of particular features. But curve which connects them by passing near them is completely different." [5]

"*This leads to method invoked by The Archaeology of Knowledge: a series continues until it passes into the neighbourhood of another individual point, at which moment another series begins, which can either converge with the firs one (statements form the same 'family') or else diverge (another family). It is in this sense that a curve carries out the relation of force by regularizing and aligning them, making the series converge, and tracing a 'general line of force' :for Foucalt, not only are curves and graph statements, but statements are kinds of curves or graphs."* [6]

This relation/non-relation cannot be reduced to a formalized function. It is impossible to simplify it to a unequivocal equivalent or to unequivocal hierarchical relation. Character of this relation can be only traced. It means that if we want to find out the state of a system or of a curve shape in a particular time, we have to go through whole process of system for the whole given time. While investigating a changing curve's attributes within the system it is not possible to disregard also an influence of other curves forces. In other words, it is not possible to gain the result using only one equation. To get the result we need to make a series of calculation in chosen time intervals. A result from this can be named as an anexact. The term anexact was introduced and described by G. Deleuze and F. Guattari in the book A Thousand plateaus. The anexact form is neither exact nor inexact. It has not been possible to simplify it into any of the 'ideal' exact geometric forms

[2] EISENMAN, Peter: Diagram Diaries.London: Thames & Hudson, 1990, P. 33.
[3] EISENMAN, Peter: Diagram Diaries.London: Thames & Hudson, 1990, P. 34.
[4] Peter Eisenmam on his lecture at The Royal Institute of British Architecture on 13 September 2004.

[5] DELEUZE, Gilles: Foucault. London: University of Minnesota Press 200, P. 78.
[6] DELEUZE, Gilles: Foucault. London: University of Minnesota Press 200, P. 78.

(as a sphere, cube or pyramid). At the same time it is not inexact, because it can be 'gained' through a series of exact techniques.
This moment is the most suitable to point out features and possibilities of animation software that enables a diagram to become visualized/materialized.

Animation software as for example Maya uses for 3D objects modelling NURBS parametric curves. In contrary to implicit curves, these parametric curves (their form, variables) are not dependent on a coordinate axis but on time. A NURBS curve is a special form of parametric curve and its form can also be influenced by the different weight of points which it passes through. This characteristic of curves is almost completely correspondent with Foucault's description of forces. This software is equipped apart from NURBS curves also with functions for definition of field intensity. Functions like these tremendously help animators to simulate real time processes, but they can also be set to any thinkable field intensity between any objects in the scene. The combination of the two mentioned possibilities of animating programmes allows to virtually build an abstract machine that Foucault writes about.

More about NURBS

What is NURBS? The Abbreviation NURBS stands for Non-Uniform Rational B-Spline.[7] It is a graphics technique for drawing a curve. NURBS is a parametric curve and is defined by a set of weighted points, the curve's order, and a knot vector. It is a generalization of B-Spline which is based on the Bézier curve, with difference being weighting or the control points which makes it rational. There are numerous advantages of using NURBS curves:

- they can accurately represent standard geometric object-like lines, circles, ellipses, spheres, as well as free form geometry, such as a car and a human body,

- they are invariant under affine as well as perspective transformations,

- the amount of information required for a NURBS representation of curved smooth geometry is much smaller than the amount of information required for common faceted approximations.

In computer graphics the two most common methods to represent curve are the implicit and the parametric method. Using the implicit method a curve is defined by a function which depends on the axis variables. In other words, they are defined as a set of every possible point, which complies with a given rule/function. For example, the function $f(x,y) = x^2+y^2-1=0$ represents a circle with the radius of 1. In the parametric method each of the axis variables is a function of an independent parameter. Using this method we are moving along the curve in time (parameter) and in every moment we define the position the of curve's point. For example, a curve would be defined with the independent variable of t

$$C(t) = [x(t), y(t)] \quad a \le t \le b$$

The parameter of t is from the given interval [a,b]. To represent the first quadrant of a circle in parametric form, we can write

$$C(t) = [\cos(t), \sin(t)] \quad 0 \le t \le 2Pi$$

Parametric representation of curve is not unique and the same quadrant of a circle can be represented parametrically also as

$$C(t) = [\,(1-t^2)/(1+t^2),\ 2t/(1+t^2)\,] \quad 0 \le t \le 1$$

Non-Uniform Rational B-Spline is one type of a parametric curve class. NURBS is used for computation, because it is easily processed by a computer, it is stable for floating points errors and has a little memory requirements. It is a generalization of non-rational B-Splines which are based on the rational Bézier curve and the rational Bézier curve is a generalization of the Bézier curve.

Bézier curves are widly used in computer graphics to model smooth curves. The curve is completely contained in the convex hull of its control points. The point can be graphically displayed and can manipulate the curve intuitively. The most important Bézier curves are quadratic and cubic. To draw more complex shapes, more low order Bézier curves are patched together under a certain smooth condition. A Bézier curve of degree n is defined by

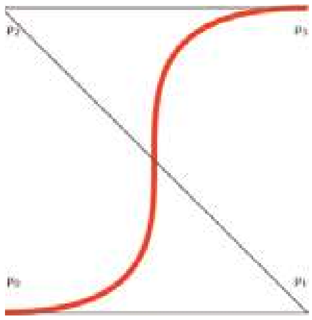
$$C(t) = \sum_{i=0}^{n} B_{i,n} P_i(t) \quad 0 \le t \le 1$$

The geometric coefficients Pi are control points and the basis functions Bi,n are the classical n-th degree Bernstein polynomials

$$B_{i,n}(t) = (n! / i!\,(n-i)!)\,t^i\,(1-t)^{n-i}$$

*n – deegree of a curve is a positive whole number. This number is usually 1,2,3 or 5. Straight lines and polylines are usually 1 degree, a circle is 2 degrees and most of the free-form curves are 3 or 5 degrees.

figure 1



[7] Spline (in mathematics) is a special curve defined piecewise by polynomials. The term spline comes from the flexible devices by shipbuilders and draftsmen to draw smooth shapes.

The Bézier curve cannot represent conic curves (a.i. a circle). A conic curve can be represented using a rational function, which is defined as a ratio of two polynomials:

$$x(t) = X(t) / (W(t) \qquad y(t) = Y(t) / (W(t) \qquad z(t) = Z(t) / (W(t)$$

According this definition, the rational Bézier curve is defined as

$$C(t)=(\sum_{i=0}^{n} B_{i,n} P_i (t) w_i) / \sum_{i=0}^{n} B_{i,n} (t) w_i \qquad 0 \le t \le 1$$

The new variables wi are scalars called weights. The varying weight value of thr control point will lead to a greater attraction or repulsion of the curve. The clearest way how to explain this is with an example. In figure 2, three Bézier curves are drawn with the only difference in the weight of the control point P2. With the weight 1.0 the shape of the curve is the same as in figure 1. The weight of 2 attracts the curve a bit more towards the control point and the weight 0.5 pushes the curve away from the point.

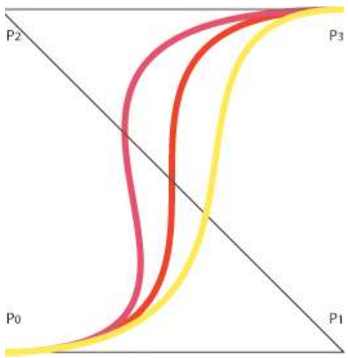


figure 2

Using only one rational Bézier curve segment for complex shapes is very difficult. The change in position of any single control point cause changes in shape in the whole curve (on all control points). It is very difficult for a user to predict them intuitively. To avoid these problems a piecewise rational curve is used.
A piecewise Bézier curve or a B-Spline (for Basic Spline) curve is constructed from several small Bézier curves of low degrees joined together. They are joint at breakpoints with a certain level of continuity.

Moving control points will affect only adjacent parts of the curve. The curve C(t) is defined in t from the interval [0,1] and it is composed of the segments C_i (t), $1 \le i \le m$. The segments are joined together at the breakpoints $t_0 = 0 < t_1 < t_2 \ldots < t_{i-1} <_{ti} =1$.
The smooth transition between two small curves can be defined in k continuity. The *k* continuity is achieved by choosing k+1 control points of the following curve according to the previous one.
The difference between the continuity of the *breakpoints* of *0* and *1* is shown in figure 3.

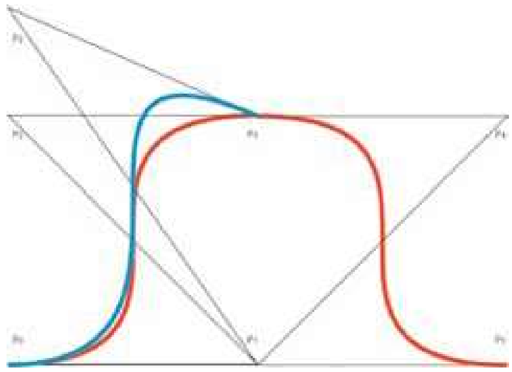


figure 3

The control points circled with another circle are control points which are used by more than one Bézier segment.
Storing this points more than ones is not memory efficient. If the curve is said to be C1 continuous , then some of the points inside a Bézier segment are dependent on the position of the previous points to satisfy the continuity constraint. Storing this point in memory is not efficient and not necessary.
The B-Spline equation is more memory efficient and also allows the local control of the curve; i.e. the basis functions are not defined over $[t_0,t_m]$. Instead, it is constrained to a limited number of subintervals.
It is defined as

$$C(u)=\sum_{i=0}^{n} N_{i,p} P_i \qquad a \le u \le b$$

P_i are the control points and $N_{i,p}$ are the p^{th} degree B-Spline functions. The B-spline has breakpoints which are called *knots*. A sequence of these knots is called a *knot vector* and it is defined as no-decreasing sequences of real numbers.
Only a rational function can represent conic curve, therefore the B-spline could be generalized to obtain rational representation. The name of this generalization is Non-Uniform Rational B-Spline (NURBS) and it is defined as

$$C^w(u)=\sum_{i=0}^{n} N_{i,p} P_i^w \qquad a \le u \le b$$

Conclusion

Reading the philosophical books of Foucault or Deleuze, theoretical works of architects, or a tutorial of animation software shows a significant correspondence in using terms as curve or spline. Apparently the terms from geometry and mathematics are very important tools for describing a diagram in Foucault's work. Comparing the two kinds of mathematic description curves, the implicit and the parametric, it is evident that the parametric curve is much more appropriate for visualizing or constructing a diagram. Considering the evolution of parametric curve forms,

Bézier curves to NURBS, the NURBS curve can be understood as the closest to the curve of a force in a diagram.
From this point of view, the use of NURBS by Gregg Lynn, and emphasizing their importance in his theoretical work appears reasonable. The only more sophisticated and appropriate computer surface than a NURBS surface would be a subdivisional surface. Unfortunately, this surface was invented after the publishing of most of Greg Lynn's books.

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